

Controllability of temporal networks: An analysis using higher-order networks

Yan Zhang, Antonios Garas, Ingo Scholtes

ETH Zürich, Chair of Systems Design
Weinbergstrasse 56/58, Zürich, Switzerland

Abstract

In this manuscript, we show how higher-order graphical models can be applied to study the controllability of networked systems with dynamic topologies. Studying empirical data on temporal networks, we specifically show that the order correlations in the activation sequence of links can both increase or reduce the time needed to achieve full controllability. We then demonstrate how spectral properties of higher-order graphical models can be used to analytically explain the effect of order correlations on controllability in temporal networks.

1 Introduction

The fundamental question if and how a system can be controlled has important applications for the study of biological, social and technical systems, as well as the development of interventions that can help us to mitigate destructive collective behavior in societies, cure diseases at a cellular level, or control distributed technical infrastructures [15]. Controlling a system means having the ability to guide the system towards a desired state with suitable signals. This is a non-trivial task, especially when interactions between the elements of a system are mediated via a complex topology. To address this problem, [16] introduced an analytical framework that combines network theory and control theory. Under the assumption that nodes follow a linear dynamics, here it has been shown that the problem of finding a minimal set of so-called *driver nodes*, which allow to control the whole system, can be cast into a graph-theoretic problem that can be solved analytically [16–18, 26, 29, 31].

While these network-based approaches have significantly advanced our ability to influence and control complex systems, the majority of existing works have assumed that the network topology is *static*, i.e. links between the elements of a system are assumed to be present and active all the time. Recent studies have not only highlighted the fact that real world complex systems exhibit time-varying topologies, they have also shown that the temporal dimension of real complex systems crucially influence dynamical processes [6, 8, 27]. These findings raise a number of important questions for the study of controllability in complex systems which have only been partly addressed recently: The authors of [22] and [20] have examined what part of the system can be controlled by means of a single driver node, finding that both the activity patterns and

degrees of nodes crucially influence controllability. Comparing a temporal network with its static counterpart, [11] have shown that the dynamic nature of links can reduce the time needed to control a system based on a fixed set of driver nodes.

While these works have already shed some light on the complex role of time-varying topologies in the control of complex systems, they have mostly focused on the non-Poissonian nature of timings of temporal interactions. However, it was recently shown that dynamical processes in temporal networks are not only driven by the *timing* of interactions, but also by the *temporal ordering* in which they occur [10, 21, 23, 25, 30]. In a nutshell, independent of how far link activations lie apart in time, whether a link (a, b) in a temporal network is activated *before* or *after* a link (b, c) crucially affects *causality*, i.e. whether node a can possibly influence node c or not. In a recent work it has been shown that the resulting effect of link ordering on dynamical processes (i) can be captured by means of studying non-Markovian characteristics in the link activation sequence, and (ii) that this effect can be understood analytically by studying *higher-order* network representations [24, 25]. Despite the importance of link ordering for dynamical processes, this effect has not been investigated systematically in the literature on controllability of complex networks, and an analytical approach to understand its effect is also absent.

Closing this gap, in this work we explore how the ordering of links in temporal networks influences the controllability of complex systems with time-varying topologies. Studying six empirical temporal networks, we first show that order correlations, captured in terms of *non-Markovian* characteristics in link activation sequences, can both increase or decrease the time to make the system fully controllable compared to a null model in which order correlations are removed. To explain this phenomenon, we extend the graphical modeling framework of *higher-order networks* introduced in [24, 25]. We specifically generalize the structural controllability framework introduced by [16] to higher-order networks, thus making it applicable to investigate the controllability of temporal networks whose link sequences are subject to order correlations. This framework not only allows to analytically explain the influence of order correlations on control, it also highlights that the controllability of temporal networks crucially depends on the complex interplay between the temporal and topological characteristics of systems. Interestingly and counterintuitively our study finally reveals that, compared to an earlier study highlighting that order correlations can either slow down or speed up diffusion processes, their effect on the time needed to control a given system can actually be the opposite.

In summary our work shows that, in order to develop a comprehensive analytical framework for controllability of networked systems, the complex interplay between temporal and topological characteristics of dynamic networks must be taken into account.

2 Controllability of temporal networks

In the following, we first introduce some basic definitions along with the key components of our methodology.

2.1 Basic definitions

We first define a temporal network or graph $G = (V, E^T)$ to consists of a set of N nodes V as well as a set $E^T \subseteq V \times V \times \mathbf{N}$ of time-stamped links $(i, j; t) \in E^T$ where $t \in \mathbf{N}$ is the discrete time stamp of a directed link from node i to j . Importantly, we assume that time-stamped links occur *instantaneously* at discrete times by default, however we can represent links persisting within a time range $[t, t + \delta]$ by including multiple time-stamped links $(i, j; \tau)$ for all $t \leq \tau \leq t + \delta$. Such a temporal network can be represented as a series of network snapshots, each snapshot at time t containing only those time-stamped links $(i, j; t)$ which occur at time t . We can further represent each snapshot by an adjacency matrix $\mathbf{A}(t) \in \mathbb{R}^{N \times N}$, where elements $a_{ij}(t)$ ($i, j = 1, \dots, N$) capture the presence of an interaction from node i to node j at time t .

To study the controllability of a temporal network, we further assume that the nodes in the temporal network follow a discrete linear dynamics, while we are free to control the *input signals* of a given set of N_d *driver nodes*. For a vector $\mathbf{X}(t) \in \mathbb{R}^N$ capturing the state of all N nodes at time t we specifically assume that the dynamics of the system is given as

$$\mathbf{X}(t+1) = \mathbf{G}(t+1)\mathbf{X}(t) + \mathbf{B}\mathbf{U}(t), \quad (1)$$

where matrix $\mathbf{G}(t+1) = [\mathbf{A}(t+1)]^T + \mathbf{I}$ captures the time-varying topology of interactions (adding self-loops). Matrix $\mathbf{B} \in \mathbb{R}^{N \times N_d}$ maps the time-varying vector $\mathbf{U}(t)$ of N_d input signals $u_j(t)$ ($j = 1, 2, \dots, N_d$) to the N_d driver nodes, i.e. we assume that $b_{ij} \neq 0$ if input signal u_j is assigned to driver node i .

It is important to highlight, that the study of dynamical processes in dynamic networks can generally involve two different timescales: A first timescale can be associated with the dynamics of the network topology, while the second timescale is associated with the evolution of the dynamical process. Importantly, with the definitions above we implicitly assume that both timescales are inherently coupled to each other, i.e. the network and the process are assumed to evolve at about the same timescale. At first glance, this assumption of a single timescale seems to simplify the problem, however we argue that the opposite is the case: If the dynamics of a process is much faster than the dynamics of the network topology, then the process can be asymptotically viewed as a process evolving in a *static* topology. Similarly, if the network changes much faster than the process evolves, the details of the network dynamics are likely to not influence the process. In consequence, we argue that the interplay between network dynamics and dynamical processes is likely to be maximal if both operate at the same timescale.

2.2 Structural Controllability Theory

We now address the controllability of the dynamical system introduced above. Following the algebraic approach introduced by Kalman [9], the size of the controllable subspace (i.e. the number of controllable nodes) of a linear dynamical system for a given set of driver nodes can be assessed by calculating the rank of a so-called *controllability matrix*. For our scenario of a dynamical system in a temporal network, the common definition of this matrix naturally leads to the following *temporal controllability matrix* [14, 20, 22]

$$\mathbf{C}_t = [\mathbf{G}_t \mathbf{G}_{t-1} \dots \mathbf{G}_1 \mathbf{B}, \mathbf{G}_t \mathbf{G}_{t-1} \dots \mathbf{G}_2 \mathbf{B}, \dots, \mathbf{G}_t \mathbf{B}, \mathbf{B}] \in \mathbb{R}^{N \times t N_d}, \quad (2)$$

where $[\mathbf{A}, \mathbf{B}]$ denotes the concatenation of two matrices \mathbf{A} and \mathbf{B} and the products $\mathbf{G}_t \cdot \dots \cdot \mathbf{G}_1$ take the role of the matrix power \mathbf{A}^t in the common definition of the controllability matrix. In general, $N_b := \text{rank}(\mathbf{C}_t) \leq N$ denotes the size of the controllable (sub)system for a given assignment of control signals to driver nodes captured in \mathbf{B} . According to the Kalman ranking condition [9], we call the system controllable if the temporal controllability matrix has full rank, i.e. all nodes can be controlled.

In general, the study of controllability based on the rank of the controllability matrix introduced in Eq.2 allows to incorporate *weighted* links, where weights capture the *strengths* of interactions between nodes. However, in many real world situations – including the data sets studied in this manuscript – the weights of links, or strengths of interactions, are unknown. This problem has been addressed based on the framework of *structural controllability* [12]. The key idea is to treat both the adjacency matrix \mathbf{A} and the “mapping” matrix \mathbf{B} as *structural matrices* whose non-zero elements are treated as free parameters. We then call a system “structurally controllable” iff we can tune the free parameters in the structural matrices \mathbf{A} and \mathbf{B} such that the rank of \mathbf{C} N_b equals N . In a recent work, [16] have shown that for static networks, structural controllability can be cast into a graph- theoretical problem.

In the following we explain how the concept of structural controllability can be generalized to temporal networks in terms of a two-step process: In the first step, we project the time-varying network topology into a so-called *time-unfolded network*, a static representation where time is “unfolded” into an additional topological dimension [21]. In the second step, we can then study the structural controllability of a temporal network by solving a graph-theoretical problem on the static, time-unfolded network.

2.3 Structural Controllability in Time-unfolded Networks

For this, we generate a time-unfolded representation of a temporal network as follows: For a given set of nodes V and time stamps $[1, \dots, T]$ we create “temporal copies” v_t for all nodes $v \in V$ and time stamps $t \in [1, \dots, T]$ as illustrated in Fig. 1. Moreover, for each time-stamped link $(v, w; t)$ we generate a directed *interaction link* (v_t, w_{t+1}) connecting the temporal copy of v at time t with the copy of w at time $t + 1$. By this, we obtain a directed acyclic graph, in which time moves from top to bottom. Moreover, this simple projection allows us to study *time-respecting paths* [19] as *static* paths in a directed acyclic graph. In addition, we introduce so-called *state persistence links* which, for each node v connect consecutive temporal copies v_t and v_{t+1} by a directed link (v_t, v_{t+1}) . As we will explain in more detail later, state persistence links (dashed links in Fig.1) ensure that the state of a node at time t is *transferred* to the next time step $t + 1$. The inclusion of such state persistence links is crucial when studying high-resolution data on temporal networks, i.e. “sparsely connected” temporal networks where only one or few links occur at each time step. Without these additional links, for such data sets the state of a temporal copy v_t would be zero whenever there are no interactions links from a previous time step. We highlight, that this assumption is different from previous works which have studied the structural controllability of time-unfolded representations of (“densely connected”) temporal networks while omitting state persistence links [22].

We finally add control signals u_k connected to all driver nodes $k = 1, 2, \dots, N_d$ at every time step t . As we shall see below, this time-unfolded projection allows us to address structural controllability of temporal networks through a static network.

In order to apply structural controllability to time-unfolded networks, let us first assume that the weights of all links, including *interaction* and *state persistence* links, were free parameters. This assumption would allow us to apply the *independent path theorem* [22], which states that a set C of nodes in the system is structurally controllable at time T , if there exist $|C|$ *independent* paths starting from any input signal to every node in C at time T . Two paths are independent if they do not pass through the same node at the same time. Based on the independent path theorem, the size N_b of the maximally controllable subsystem equals the maximum number of independent paths.

Importantly, the assumption that *all* link weights are free parameters introduces the problem that we could assign weights to state persistence links, which we merely introduced to “copy” the state of a nodes to the next time step. If we set the weight of these persistence links to values larger or smaller than one, this would introduce an amplification or dampening of the state, even in absence of an interaction with other nodes. Specifically, if state persistence links are omitted as in [22], the state of nodes will immediately fall back to zero in absence of external interactions. The need to fix the weights of all persistence links in the time-unfolded network to

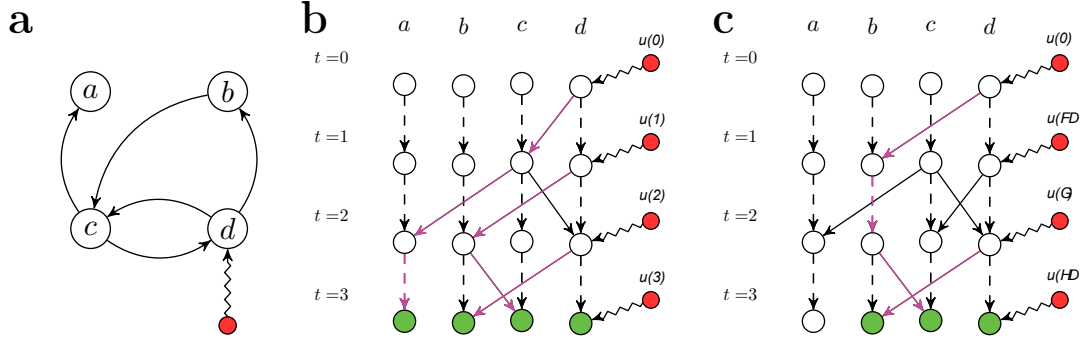


Figure 1: Controlling a simple time varying system of four nodes with only one input signals. (a) Aggregated representation of the system. (b-c) Time-unfolded representations of two temporal networks with the same aggregate topology as in panel (a), but different orders of links. Red nodes denote control signals. In each of the two temporal networks, we highlight one maximal set of independent time respecting paths in purple, and highlight the corresponding controlled subsystem in green. In panel (b), every node is controllable at time $t = 3$, so controllability of the whole system is achieved after three time steps. As comparison, only three nodes are controllable at time $t = 3$ in panel (c). The difference in the size of the controllable (sub)system shows that the temporal ordering of links affects the controllability of time varying systems.

one while varying weights of interaction links implies that (i) state persistence links are *not* free parameters, and (ii) we can thus not directly apply structural controllability theory to calculate the size of the controllable subsystem.

To overcome this problem, we adapt the structural controllability framework in such a way that it (i) accounts for the special semantics of *state persistence links* and *temporal copies* in time-unfolded representations of temporal networks. We first note that, to be able to control the state of all nodes at time T , we do not need to control all *temporal copies* of these nodes in the time steps $t < T$. In other words, all nodes in a temporal network can be controllable at time t even though not all nodes (and temporal copies of nodes) in the *time-unfolded* network are controllable.

To check whether a set of nodes in a temporal network is controllable at time T , we can apply the notion of *stem-cycle disjoint subgraphs* as used in [16]. Here, a so-called stem is any sequence of links that originates from an input signal, and a cycle is a sequence of links in which the first and the last node coincide. The stem-cycle disjoint subgraph of a graph contains only those stems and cycles where each node belongs either to exactly one stem or to exactly one cycle. As shown in [16], for a static graph every node in a stem-cycle disjoint subgraph is structurally controllable.

In our case the time-unfolded network is a static, acyclic representation of a temporal network. As a result, the stem-cycle disjoint subgraph for a time-unfolded network is just a set of disjoint stems and has no cycles. As mentioned above, all nodes v in a set C can be controllable at time T , even though the temporal copies v_t of these nodes at time $t < T$ are not controllable. As such, here we are only interested in the question whether we can find a set of disjoint stems such that each node v_T in C is the end of a stem. Notably, this set of *disjoint* stems corresponds to a set of *independent* time-respecting paths between driver nodes and the nodes in C , i.e. a set of time-respecting paths whose nodes are not overlapping.

Fig. 1 illustrates the notion of controllability in temporal networks, as well as its dependence to the structure of independent time-respecting paths. Fig. 1 (b) and (c) contain time-unfolded representations of two different temporal networks which are consistent with the same time-aggregated topology shown in Fig. 1 (a). In this example, we are interested in the controllability of the four nodes at time $T = 3$, considering a single driver node d receiving a time-varying input signal u_t . Links that belong to an independent time-respecting path from this driver node to one of the temporal copies at $T = 3$ are highlighted in purple. In Fig. 1 (b), all four temporal copies at time $T = 3$ are on the end of independent time-respecting paths and thus the whole system can be controlled at time three. In contrast, in Fig. 1 (c), only three of the four nodes can be controlled, while node a is not at the end of an independent time-respecting path from a driver node. Since the temporal networks in Fig. 1 (b) and (c) have the same time-aggregated topology, this simple example further highlights the influence of the ordering links on controllability in temporal networks.

Having illustrated our approach, we finally explain how we can efficiently calculate the controllable system size N_b by identifying the maximum number of independent paths in a time-unfolded network. The procedure works by constructing an auxiliary network H as shown in Fig. 2 (b). First, we replace each node v except for driver nodes with v_{out} and v_{in} . (see Fig. 2(a)) where v_{in} collects all links pointing to v while v_{out} collects all links originating from v . We further include an additional link from each v_{in} to the corresponding v_{out} . This node-splitting procedure reflects the constraint that two paths can not pass through the same node v if we set the weight of this additional link to 1. Moreover, we add one *source node* which is connected via directed links to all input signals at all time steps. Finally, we add one *sink node* along with directed links connecting all temporal copies at time T to this sink node. The result is the auxiliary network H presented in Fig. 2 (b). Based on this construction, the task of finding a maximum set of independent time-respecting paths corresponds to identifying a maximum flow from source to sink in the auxiliary network where all link capacities are set to one[?]. These link capacities of one capture the constraint that only one path is allowed to pass through one node at a given time. With this, the size of the controllable subsystem N_b at time T corresponds to the maximum flow from source to sink, which can be easily solved in polynomial time [7].

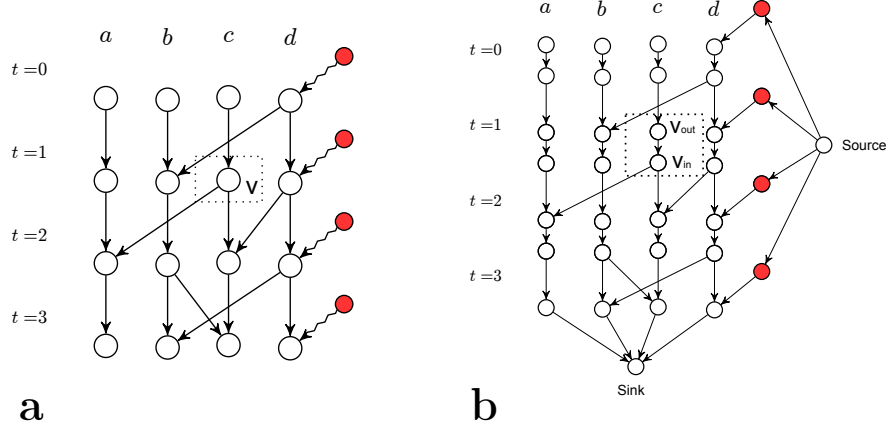


Figure 2: Illustration of the auxiliary time-unfolded network to identify the maximum number of independent time-respecting paths. This illustration shows the case where an input signal is attached to only one driver node, however the same construction applies to cases with multiple driver nodes.

2.4 Empirical Results

We now apply the method introduced above to study how the temporal ordering of links affects structural controllability in temporal networks. We test our method on the following six empirical data sets describing different kinds of temporal networks: (AN) describes 1,911 pairwise interactions between 89 ants in a colony [3]. (RM) contains 26,260 interactions among 64 students and academic staff in a university campus [4]. (EM) captures 11,000 e-mails exchanged between 167 employees in a manufacturing company for one month [5]. (HO) contains more than 15,000 time-stamped contacts recorded by proximity sensing badges among 46 healthcare workers and 29 patients in a hospital for 48 hours [28]. (FL) includes 230,000 multi-segment flights among 116 US airports in the fourth quarter of 2001 [1]. (LT) contains itineraries of passengers using the 309 London Underground stations, extracted from the Rolling Origin and Destination Survey database that covers four million passenger flows for one week [2]. Note that these six empirical data sets have been used in [25], which also includes a detailed technical description of how they have been processed.

To explore to what extent the time ordering influences controllability, we compare each of the empirical data sets with a randomized version, in which we shuffle the time stamps of all interaction events. By this, we remove correlations in the ordering of interaction events, while the resulting null model has the same time-aggregated network as the empirical data set. To quantify the effect of link ordering on controllability, we calculate the relative size of the sub-

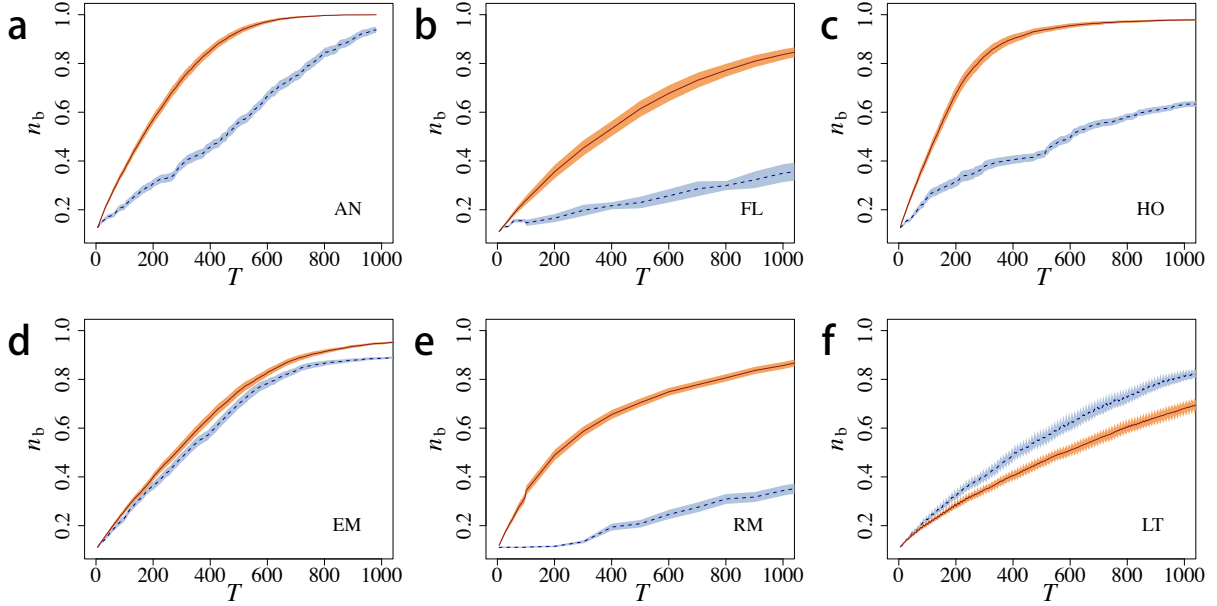


Figure 3: Relative size of the controllable system $n_b(T)$ at time T , where a random sample of 5% of nodes are used as driver nodes. Blue dashed lines correspond to the original interaction sequences, the orange continuous lines correspond to shuffled interaction sequences. The shaded areas indicate the 95% confidence intervals of $n_b(T)$ for 100 realizations.

system $n_b(T) = N_b(T)/N$ that can be controlled at a given time T both for the empirical and the shuffled data. To facilitate the comparison, we use the same set of randomly sampled driver nodes for each time step, i.e. we choose a set of driver nodes once and then calculate the size of the controllable sub system for (i) the empirical sequence, and (ii) one shuffled sequence for all times T . We then repeat this procedure 100 times. For the results in this manuscript, we have chosen a fixed fraction of driver nodes of 5%, however our simulations show that our results do not qualitatively depend on this choice.

Fig. 3 shows the relative size $n_b(T)$ of the subsystem controllable at time T for each of the six data sets along with the corresponding shuffled versions. The fact that the 95% confidence interval is barely visible for all six cases in Fig. 3 confirms that the choice of driver nodes does not strongly influence our results. Whenever $n_b(T)$ for the empirical data set is smaller than for the randomized version at a given T , the ordering of links in the interaction sequence negatively affects the size of the controllable subsystem, and vice versa. Our results show that for all six data sets time ordering of interactions significantly influences controllability. More precisely, for five out of six cases the size of the controllable subsystem grows slower due to correlations in the temporal ordering, while for one case (LT) these correlations actually speed up controllability.

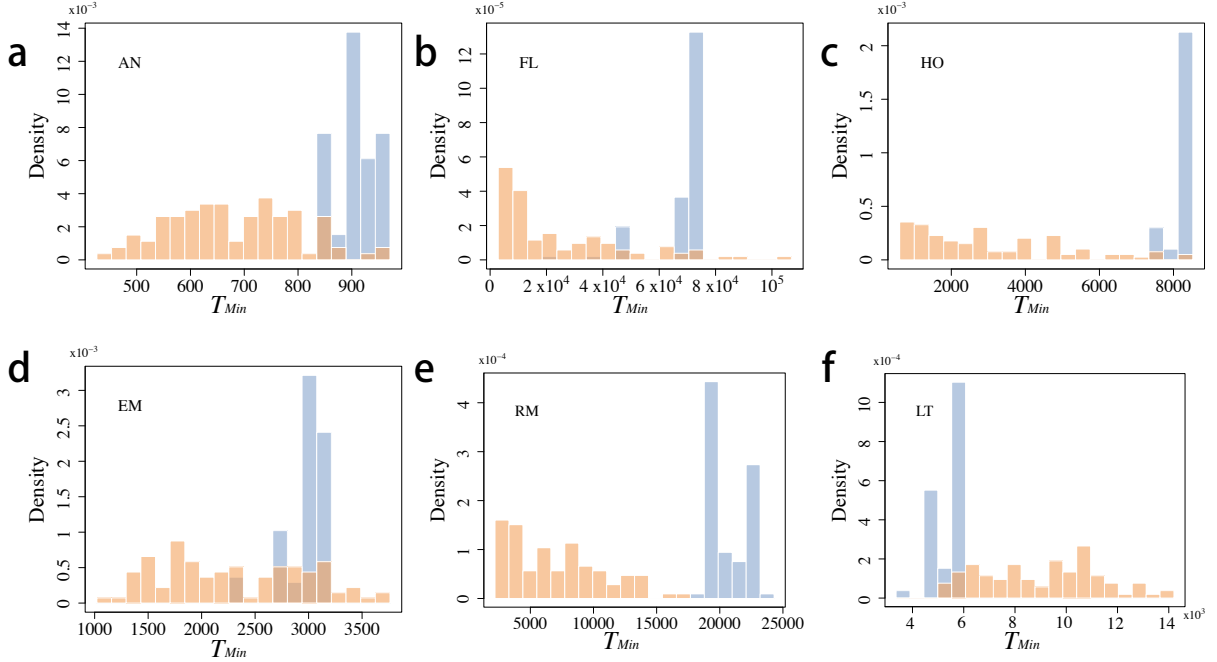


Figure 4: Distribution of the minimum time T_{Min} required to achieve controllability of the whole system (i.e. $n_b(T_{Min}) = 1$). The blue bars refer to the original interaction sequences, and the orange bars to the shuffled interaction sequences.

Besides the size of the controllable subsystem at a given time T , we can further investigate the minimum time T_{Min} required to control the full system, i.e. $T_{Min} := \arg \min_T n_b(T) = 1$ Fig. 4 compares the distribution of T_{Min} for each of the real data sets with its shuffled counterpart. As before, for five out of the six cases the peak of the T_{Min} distribution in the shuffled sequence is shifted to the right compared to the empirical data, thus indicating that the ordering of interactions slows down controllability. On the other hand, for (LT) the peak of the distribution for the shuffled sequence is shifted to the left compared to the empirical data, thus indicating a salient speed-up of controllability. These results show that the time ordering of interactions in temporal networks can have both a speed-up and slow-down effect on controllability. It further raises the question how this non-trivial effect on controllability can be explained and predicted.

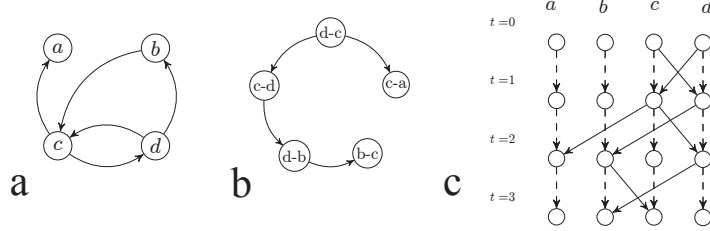


Figure 5: (a) First-order and (b) second-order aggregated presentation of a temporal network consisting of four nodes and four time steps (c).

2.5 Higher-order representation of temporal networks

To address this question, we deploy the higher-order abstractions of temporal networks introduced in [25]. The key motivation for this abstraction is to construct a static, time-aggregated representation of temporal networks that encodes information on both the topology and the temporal ordering of time-stamped links. To facilitate our analysis, here we define second-order aggregated representations of temporal networks, which is the simplest possible generalization of the commonly used (first-order) time-aggregated networks [24, 25]. A first-order time-aggregated network is constructed by aggregating all links (interactions) that occur in a temporal network. Intuitively, the weight of a link in this static representation can be defined as the number of times it appears in the temporal network. Building on the observation, that the weights of links in such *first-order* time-aggregated representations count the frequency of links – and thus time-respecting path of length one – we can generalize this approach to account for higher-order path lengths. As such, a *second-order* time-aggregated network can be constructed following a line graph construction: First of all, each link (a, b) in the first-order network defines a node $a - b$ in the second-order network. Two second-order nodes $a - b$ and $b - c$ are connected by a directed second-order link $(a - b, b - c)$, iff the corresponding time-respecting path $a \rightarrow b \rightarrow c$ of length two exists. Moreover, we define the weight of the link $(a - b, b - c)$ to capture the frequency of the time-respecting path $a \rightarrow b \rightarrow c$ in the temporal network. An example for this construction is shown in Fig. 5, which shows both the first- (b) and the second-order (c) time-aggregated representation of a temporal network (d). As has been shown in [24, 25] a key benefit of such a higher-order representation is, that it better captures the *causal topology* of temporal networks, i.e. who is connected to whom in terms of time-respecting paths.

In particular, previous studies have shown that the effect of the ordering of links on the causal topology can be understood analytically based on the algebraic properties of higher-order net-

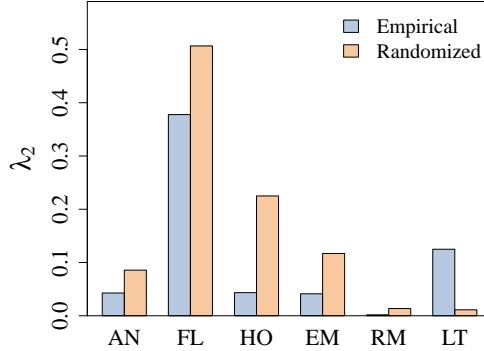


Figure 6: Algebraic connectivity λ_2 of the second-order network of the empirical temporal networks (blue) and a shuffled temporal sequence without order correlations (orange).

works [21, 25]. Interpreting controllability as a propagation of independent control signals, in the following we follow a similar approach by quantitatively studying how “connected” the causal topology of a temporal network is. Intuitively, the “more connected” a temporal network is, the faster the propagation of control signals and the faster we can control the system. This level of connectivity is captured by the algebraic connectivity of the network topology, which is defined as the second-smallest eigenvalue, λ_2 , of its Laplacian matrix. We thus hypothesize that the slow-down or speed-up of controllability observed in the empirical data sets can be explained by changes in the algebraic connectivity of second-order networks that are due to the ordering of links.

Fig. 6 compares the algebraic connectivity λ_2 of the second-order time-aggregated network for each of the empirical data sets with the algebraic connectivity of a shuffled counterparts. We observe that for the five cases where we observed a slow-down in controllability, λ_2 for the empirical network is smaller than for its shuffled counterpart. For the (LT) data set, which is the only case in which we observed a speed-up, λ_2 for the empirical network is much larger than for the shuffled version. In consequence, a comparison of the algebraic connectivity of a second-order representation of a temporal network with its shuffled counterpart provides us with a simple way to qualitatively predict how the ordering of links affects controllability.

As a final remark, we contrast our findings to the results presented in [25], which used a similar algebraic approach to study the effect of link ordering on the speed of a diffusion process. Interestingly, our results highlight that the effect of link ordering on diffusion dynamics and control can be different in the same data set. In particular, our study reveals that the ordering of links in the (FL) data set slows down controllability, while in [25] it has been shown that it speeds up diffusion. These opposite effects can be intuitively understood by considering that the speed of diffusion is related to the convergence time of a random walker, while the emergence of controllability is related to the time at which nodes are first reached by a control signal. From

an algebraic point of view, this intuition is captured by the fact that the *speed up* of diffusion in (LT) can be analytically explained based on the spectral gap of a transition matrix [25], which captures the convergence time of a random walker. In contrast, in our work we have shown that the *slow down* of controllability can be predicted based on the algebraic connectivity, which has been shown to capture the first hitting time of a random walker [13].

3 Discussion

In summary, we explored how the ordering of links affects controllability of temporal networks. We further showed how structural controllability theory can be extended to study the size of the controllable subsystem based on time-unfolded networks with state persistence links. We particularly argue that the inclusion of state persistence links in time-unfolded networks prevents a direct application of structural controllability theory. The reason for this is that, if the weights of state persistence links are fixed values of ones, the time-unfolded network cannot be studied as a structural matrix where all non-zero entries are free parameters. In our work, we showed that (i) the weights of state persistence links do not influence the controllability of a temporal network, and (ii) the size of the controllable subsystem can be calculated by solving a maximum flow problem on an adjusted time-unfolded network.

We applied our method to six empirical data sets capturing temporal networks from different contexts. A comparison with shuffled versions in which all order correlations are destroyed reveals that the ordering of links in a temporal network can significantly speed up or slow down controllability. Furthermore, by making a link to the study of dynamical processes on temporal networks [25], we demonstrated that we can explain the effect of time ordering of interactions on controllability based on the algebraic connectivity of higher-order aggregate representations of temporal networks. Counter-intuitively, we found that the effect of link ordering on controllability and diffusion can be opposite even for the same data set.

4 Acknowledgements.

We gratefully acknowledge discussions with Frank Schweitzer (Chair of Systems Design, ETH Zürich) and Yuan Lin (School of Computer Science, Fudan University) on an early draft of this work. We are further grateful for helpful comments by Yang-Yu Liu (Harvard Medical School). A.G. acknowledges support from the EU-FET project MULTIPLEX 317532. I.S. acknowledges support from the Swiss State Secretariat for Education, Research and Innovation (SERI), Grant No. C14.0036 and from the MTEC Foundation in the context of the project “The Influence of Interaction Patterns on Success in Socio-Technical Systems”.

References

- [1] Bureau of Transportation Statistics. RITA TranStat Origin and Destination Survey database, 2014.
- [2] Transport for London. Rolling Origin and Destination Survey (RODS) database., 2014.
- [3] Benjamin Blonder and Anna Dornhaus. Time-ordered networks reveal limitations to information flow in ant colonies. *PLoS ONE*, 6(5), 2011.
- [4] Nathan Eagle and Alex Pentland. Reality mining: Sensing complex social systems. *Personal and Ubiquitous Computing*, 10(4):255–268, 2006.
- [5] Series Editors. *Lecture Notes in Business Information Processing*. 2010.
- [6] Kwang-Il Goh and Albert-László Barabási. Burstiness and Memory in Complex Systems. *ArXiv*, 0(1):4, 2006.
- [7] Andrew V. Goldberg and Robert E. Tarjan. A new approach to the maximum-flow problem. *Journal of the ACM*, 35(4):921–940, 1988.
- [8] Hang Hyun Jo, Marton Karsai, Janos Kertesz, and Kimmo Kaski. Circadian pattern and burstiness in mobile phone communication. *New Journal of Physics*, 14, 2012.
- [9] R E Kalman. Mathematical Description of Linear Dynamical Systems. *J.S.I.A.M. Control*, 1(2):152–192, 1963.
- [10] Renaud Lambiotte, Vsevolod Salnikov, and Martin Rosvall. Effect of memory on the dynamics of random walks on networks. *Journal of Complex Networks*, 2014.
- [11] Aming Li, Sean P Cornelius, Yang-yu Liu, and Long Wang. The fundamental advantages of temporal networks. *arXiv:1607.06168v1*, pages 1–45.
- [12] Ching Tal Lin. Structural Controllability. *IEEE Transactions on Automatic Control*, 19(3):201–208, 1974.
- [13] Yuan Lin and Zhongzhi Zhang. Random walks in weighted networks with a perfect trap: An application of Laplacian spectra. *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics*, 87(6), 2013.
- [14] Xiaomeng Liu, Hai Lin, and Ben M. Chen. Structural controllability of switched linear systems. *Automatica*, 49(12):3531–3537, 2013.
- [15] Yang-Yu Liu and Albert-László Barabási. Control principles of complex systems. *Rev. Mod. Phys.*, 88:035006, Sep 2016.

- [16] Yang-Yu Liu, Jean-Jacques Slotine, and Albert-László Barabási. Controllability of complex networks. *Nature*, 473(7346):167–73, 2011.
- [17] Giulia Menichetti, Luca Dall’Asta, and Ginestra Bianconi. Control of Multilayer Networks. *Scientific Reports*, 6:20706, 2016.
- [18] Tamás Nepusz and Tamás Vicsek. Controlling edge dynamics in complex networks. *Nature Physics*, 8(7):568–573, 2012.
- [19] Raj Kumar Pan and Jari Saramäki. Path lengths, correlations, and centrality in temporal networks. *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics*, 84(1), 2011.
- [20] Yujian Pan and Xiang Li. Structural controllability and controlling centrality of temporal networks. *PLoS ONE*, 9(4):123055, 2014.
- [21] René Pfitzner, Ingo Scholtes, Antonios Garas, Claudio J. Tessone, and Frank Schweitzer. Betweenness preference: Quantifying correlations in the topological dynamics of temporal networks. *Phys. Rev. Lett.*, 110:198701, May 2013.
- [22] Márton Pósfai and Philipp Hövel. Structural controllability of temporal networks. *New Journal of Physics*, 16, 2014.
- [23] Martin Rosvall, Alcides V Esquivel, Andrea Lancichinetti, Jevin D West, and Renaud Lambiotte. Memory in network flows and its effects on spreading dynamics and community detection. *Nature communications*, 5, 2014.
- [24] Ingo Scholtes, Nicolas Wider, and Antonios Garas. Higher-order aggregate networks in the analysis of temporal networks: path structures and centralities. *European Physical Journal B*, 89(3):1–15, 2016.
- [25] Ingo Scholtes, Nicolas Wider, René Pfitzner, Antonios Garas, Claudio Juan Tessone, and Frank Schweitzer. Causality-driven slow-down and speed-up of diffusion in non-Markovian temporal networks. *Nature communications*, 5:5024, 2014.
- [26] Jie Sun and Adilson E. Motter. Controllability transition and nonlocality in network control. *Physical Review Letters*, 110(20), 2013.
- [27] Sarah Touati, Mark Naylor, and Ian G. Main. Origin and nonuniversality of the earthquake interevent time distribution. *Physical Review Letters*, 102(16), 2009.
- [28] Philippe Vanhems, Alain Barrat, Ciro Cattuto, Jean-Francois Pinton, Nagham Khanafer, Corinne Regis, Byeul-a Kim, Brigitte Comte, and Nicolas Voirin. Estimating potential infection transmission routes in hospital wards using wearable proximity sensors. *PLoS ONE*, 8(9):e73970, 2013.

- [29] Zhengzhong Yuan, Chen Zhao, Wen Xu Wang, Zengru Di, and Ying Cheng Lai. Exact controllability of multiplex networks. *New Journal of Physics*, 16, 2014.
- [30] Yan Zhang, Lin Wang, Yi-Qing Zhang, and Xiang Li. Towards a temporal network analysis of interactive WiFi users. *EPL (Europhysics Letters)*, 98(6):68002, 2012.
- [31] Chen Zhao, Wen-Xu Wang, Yang-Yu Liu, and Jean-Jacques Slotine. Intrinsic dynamics induce global symmetry in network controllability. *Scientific reports*, 5:8422, 2015.